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Applications of Sphere Packing Theory for Determining Optimal Structure of Livestock

Altankhuyag D[1*], Tungalag N[2], and Enkhbat R[2]

[1]*Business and Information technology school, National University of Mongolia*

[2]*Business School, National University of Mongolia*

[*]Corresponding author:

Altankhuyag D, Business and Information technology school, National University of Mongolia.

Abstract

The application of economic–mathematical models in the livestock sector plays a crucial role in reducing risks, lowering costs, and supporting efficient decision-making as well as in optimizing management practices and enhancing overall production performance.

Through effective management and strategic planning of herd structure and turnover, production output and economic efficiency can be stabilized.

The objective of this study is to develop a mathematical model describing the optimal structure and turnover of livestock herds based on sphere packing theory. Official statistical data on herd size, composition, and turnover were utilized. By incorporating structural and turnover constraints of livestock herds, a mathematical model of herd structure and dynamics was formulated, and simulation analysis were conducted to evaluate the model.

The structure and turnover of the livestock herd were derived using herd balance equations based on sphere packing theory [1].

The proposed approach can be used as an advanced management tool at the national, regional, provincial, and household levels.

Zavkhan Province of Mongolia was selected as a case study to examine the applicability of the proposed model to sheep herding.

Keywords: Sheep Farming, Herd Structure, Herd Dynamics, Economic–mathematical Model, Sphere Packing Theory.

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Introduction

The livestock sector constitutes an essential component of the economy, furnishing raw materials to the processing industry and ensuring food provision for the population.

The livestock sector is highly dependent on natural and climatic conditions, and disasters can lead to consequences such as livestock population decline, reduced fertility, increased animal diseases, and loss of herd structure while also leading to increased poverty and unemployment in society and economy, a decline in the standard of living of the people, and an impact on the industrial production through creating economic instability.

As a prerequisite for developing a livestock growth model and calculating the optimal herd structure, some research was conducted on the types of livestock, numbers, and structure of sheep herds in Zavkhan province over the past 5 years.

Livestock herd structure is subject to change under the influence of numerous internal and external factors. In livestock production planning, herd structure and turnover play a

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critical role. The scholarly contributions of research papers [2-4] have significantly advanced the field of mathematical modeling in animal husbandry and continue to play a pivotal role in the development of Mongolia's livestock production sector. Livestock income maximization problem has been considered in [5]. Unlike, in this paper we examine livestock structure and its income from a view point of sphere packing theory [5].

Methodology

In this paper, we propose a new application of "Sphere packing theory" which can be used in determining optimal structure of livestock.

The sphere packing theory deals with packing none overlapping spheres of the maximum volume in a given set [1]. In order to examine optimal structure of livestock from a view point of sphere packing theory it is worth mentioning the latest result on this theory.

Let $B(x0, r)$ be a ball with a center $x^0 \in R^n$ and radius $r \in R$.
$B(x^0, r) = \{x \in Rn \| x - x^0 \| \le r\}$,

Here <,> denotes the scalar product of two vectors in R^n, and $\|\cdot\|$ is Euclidean norm. The n-dimensional volume of the Euclidean ball $B(x^0, r)$ is [1]

$$V(B) = \frac{\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2}+1)} r^n,$$

where, Γ is Leonhard Euler's gamma function. Define the set of D as follows.

$$D = \{x \in R^n \mid \langle a^i, x\rangle \le b_i, i = \overline{1,m}\},$$

where, $a^i \in R^n$, $bi \in R$, $i = \overline{1,m}$.

Assume that D is a compact set and $D \neq \emptyset$. Clearly, D is a convex set in R^n.

Lemma [1]. B(x0, r)$\subset$ D if and only if

$$\langle a^i, x\rangle + r\|a^i\| \le b_i, i = \overline{1,m}.$$

$$r \ge 0.$$

Then problem of inscribing the largest sphere into D reduces to the following linear programming.

$$\max r \quad (1)$$

subject to

$$\langle a^i, x\rangle + r\|a^i\| \le b_i, i = \overline{1,m}, \quad (2)$$

$$r \ge 0. \quad (3)$$

Let (x^0, r^0) be a solution to the above problem and if we take any

$$h = (h1, h2, \dots, h_n) \in R^n$$

then,

$$x_t^h = (x_{1,t}, \dots, x_{n,t}) = \left(x_{1,t}^0 + r_t^0 \frac{h_1}{\|h\|}, \dots, x_{i,t}^0 + r_t^0 \frac{h_i}{\|h\|}, \dots, x_{n,t}^0 + r_t^0 \frac{h_n}{\|h\|}\right)$$

$$\bar{x}_t^h = (\bar{x}_{1,t}, \dots, \bar{x}_{n,t}) = \left(x_{1,t}^0 - r_t^0 \frac{h_1}{\|h\|}, \dots, x_{i,t}^0 - r_t^0 \frac{h_i}{\|h\|}, \dots, x_{n,t}^0 - r_t^0 \frac{h_n}{\|h\|}\right)$$

Clearly $x_t^h, \bar{x}_t^h \in B(x^0, r)$, consequently $x_t^h, \bar{x}_t^h \in D$.

According to sphere packing theory, a feasible point x_t must satisfy the following conditions:

$$x_{j,t}^0 - r_t^0 \frac{h_{j,t}}{\|h\|} \le x_{j,t} \le x_{j,t}^0 + r^0 \frac{h_{j,t}}{\|h\|}, \text{ j=1,…,n.}$$

Dynamic Model of Herd Turnover

We write the herd turnover balance equations for each period of t to determine the optimal structure and turnover of a sheep herd. The livestock dynamics can be expressed by the following equations.

Annual turnover:

1. Annual turnover of each age and sex group within the herd:
$x_{j,t} \le b_{j,t} + x_{j+9,t} + x_{j+15,t} - x_{j+24,t} - x_{j+33,t} - x_{j+42,t} - x_{j+51,t}, \ j = 1,2,\dots,9$
2. Ratio of ewes to lambs:
$q_t \cdot (x_{2,t} + x_{5,t}) \le n_1 + n_2 + n_3$
3. Sex ratio of lambs: $(n_{1,t} + n_{2,t}) = e \cdot n_{3,t}$
4. The number of lambs /young animals/ reared at the end of the year:
$$\sum_{j=7}^{9} x_{j,t} = (n_{1,t} + n_{2,t} + n_{3,t}) + \sum_{j=7}^{9}(-x_{j+15,t} + x_{j+24,t} + x_{j+33,t} + x_{j+42,t} - x_{j+51,t})$$

Age progression:

5. Age progression from young animals /year-old-lamb/ to adult sheep and from lamb to year-old-lamb
$x_{j+6,t} - x_{j+15,t} + x_{j+24,t} + x_{j+33,t} + x_{j+42,t} + x_{j+51,t} = x_{j,t}, \ j = 4,5,6,7,8,9$

Age and gender relationship:

6. Matching the number of ewes and rams:
$k_1 \cdot (x_{1,t} + x_{4,t}) \le (x_{2,t} + x_{5,t}) \le k_2 \cdot (x_{1,t} + x_{4,t})$
7. The proportion of ewes in the total herd, (%)
$$n \cdot \sum_{j=1}^{9} x_{j,t} \le (x_{2,t} + x_{5,t})$$
8. Matching of the number of rams and year-old rams:
$$z_1 \cdot x_{1,t} \le x_{4,t} \le z_2 \cdot x_{1,t}$$
9. The number ratio of male sheep and year-old-male lambs:
$$w_1 \cdot x_{3,t} \le x_{4,t} \le w_2 \cdot x_{3,t}$$
10. The matching number female sheep and year-old female sheep:
$$v_1 \cdot x_2 \le x_{5,t} \le v_2 \cdot x_2$$
11. Sales-Slaughtering limit:
$$h_{j,t} \le x_{j+33,t} + x_{j+42,t} \le H_{j,t}, \ j = 1,2,\dots 9$$
12. Lower and upper limits of abnormal loss:
$$m_{j,t} \le x_{j+51,t} \le M_{j,t}, \quad j = 1,2,3,\dots,9$$
or abnormal loss can be calculated by average annual percentage $x_{j,t} = A_{j,t}$, $J = 1,2, \dots ,9$
13. The number of livestock at the end of the year: $L_{t+1} \ge g \cdot L_t$, g – growth for period t, here, $x_{1,t} + x_{2,t} + x_{3,t} + x_{4,t} + x_{5,t} + x_{6,t} + x_{7,t} + x_{8,t} + x_{9,t} = L_t$.
14. Nonnegative constraints for all variables: $x_{j,t} \ge 0$, $j = 1,2, \dots ,9$
15. Amount of income from sheep wool
$$Z = \sum_{j=1}^{9} x_{j,t} \cdot q_j \cdot p_{j,k}$$

The following parameters are used with notations:

- $b_{j,t}$ - *the number of sheep in the j-th age–sex group at the beginning of year t.*
- $x_{j,t}$ - *the number of sheep in the j-th age–sex group at the end of year t.*

- q_t - *denotes the number of lambs raised per 100 ewes in the j-th age–sex group in year t.*
- *et - denotes the sex ratio of lambs in year t.*
- k_1, k_2 - *the upper and lower limits of the number of ewes per ram.*
- z_1, z_2 – *the upper and lower limits of the replacement rate of rams with year-old rams.*
- w_1, w_2 - *the upper and lower limits of the replacement rate of ewes with year-old ewe lambs.*
- v_1, v_2 - *the upper and lower limits of the replacement rate of male sheep with year-old ram lambs.*
- $h_{j,t}$ *and* $H_{j,t}$ - *the lower and upper limits of sales and slaughtering in the j-th age– sex group in year t.*
- $m_{j,t}$ *and* $M_{j,t}$ - *the lower and upper limits of abnormal losses for the j-th age and sex group in year t.*
- L_t – *the sheep flock size in year t, or the forecast value.*
- $q_{j,k}$ - *the yield norm of product type k obtained from sheep in age–sex group j.*
- $p_{j,k}$ - *the market price per unit of k-type product obtained from sheep in the j-th age–sex group.*
- z – income from sheep wool
- γ_0 - a minimum income level.

In the calculation of the economic–mathematical model for determining the structure and turnover of a sheep herd, the following auxiliary table was used, as well as official statistical data on the productivity and prices of Mongolian sheep wool.

Then, a problem of finding optimal structure of livestock to subject to constraints of a minimum level income of γ_0 is the following:

$$Z = \sum_{j=1}^{9} x_{j,t} \cdot q_j \cdot p_{j,k} \geq \gamma_0 \qquad [4]$$

$$x_{j,t} \leq b_{j,t} + x_{j+9,t} + x_{j+15,t} - x_{j+24,t} - x_{j+33,t} - x_{j+42,t} - x_{j+51,t}\ , \quad j = 1,2,\dots,9 \qquad [5]$$

$$q_t \cdot (x_{2,t} + x_{5,t}) \leq n_1 + n_2 + n_3 \qquad [6]$$

$$(n_{1,t} + n_{2,t}) = e \cdot n_{3,t} \qquad [7]$$

$$\sum_{j=7}^{9} x_{j,t} = (n_{1,t} + n_{2,t} + n_{3,t}) + \sum_{j=7}^{9}(-x_{j+15,t} + x_{j+24,t} + x_{j+33,t} + x_{j+42,t} - x_{j+51,t}) \qquad [8]$$

$$x_{j+6,t} - x_{j+15,t} + x_{j+24,t} + x_{j+33,t} + x_{j+42,t} + x_{j+51,t} = x_{j,t}, \quad j = 4,5,6,7,8,9 \qquad [9]$$

$$k_1 \cdot (x_{1,t} + x_{4,t}) \leq (x_{2,t} + x_{5,t}) \leq k_2 \cdot (x_{1,t} + x_{4,t}) \qquad [10]$$

$$n \cdot \sum_{j=1}^{9} x_{j,t} \leq (x_{2,t} + x_{5,t}) \qquad [11]$$

$$z_1 \cdot x_{1,t} \leq x_{4,t} \leq z_2 \cdot x_{1,t} \qquad [12]$$

$$w_1 \cdot x_{3,t} \leq x_{4,t} \leq w_2 \cdot x_{3,t} \qquad [13]$$

$$v_1 \cdot x_2 \leq x_{5,t} \leq v_2 \cdot x_2$$

$$h_{j,t} \leq x_{j+33,t} + x_{j+42,t} \leq H_{j,t}\ ,\ \ j = 1,2,\dots 9 \qquad [14]$$

$$m_{j,t} \leq x_{j+51,t} \leq M_{j,t}\ , \quad j = 1,2,3,\dots,9 \qquad [15]$$

$$x_{j,t} = A_{j,t}\ ,\ \ j = 1,2,\dots,9 \qquad [16]$$

$$L_{t+1} \geq g \cdot L_t\ , \qquad [17]$$

$$x_{1,t} + x_{2,t} + x_{3,t} + x_{4,t} + x_{5,t} + x_{6,t} + x_{7,t} + x_{8,t} + x_{9,t} = L_t. \qquad [18]$$

$$x_{j.t} \geq 0,\ \ j = 1,2,\dots,9 \qquad [19]$$

Now if we apply sphere packing approach to system (4)-(19), then we have the following linear programming problem [1].

$$\max r \qquad (20)$$

$$x_{j,t} - x_{j+9,t} - x_{j+15,t} + x_{j+24,t} + x_{j+33,t} + x_{j+42,t} + x_{j+51,t} + \sqrt{7} \cdot r < b_{j,t},\ \ j = \overline{1,9} \qquad (21)$$

$$q_t \cdot x_{2,t} + q_t \cdot x_{5,t} + q_t \cdot \sqrt{2} \cdot r < \sum_{i=1}^{3} n_i \qquad (22)$$

$$(n_{1,t} + n_{2,t}) = e_t \cdot n_{3,t} \qquad (23)$$

$$\sum_{j=7}^{9}(x_{j,t} + x_{j+15,t} - x_{j+24,t} - x_{j+33,t} - x_{j+42,t} + x_{j+51,t}) + \sqrt{18} \cdot r < \sum_{i=1}^{3} n_i \qquad (24)$$

$$x_{j+6,t} - x_{j+15,t} + x_{j+24,t} + x_{j+33,t} + x_{j+42,t} + x_{j+51,t} - x_{j,t} + \sqrt{7} \cdot r < 0,\ j = \overline{1,9} \qquad (25)$$

$$k_1 \cdot x_{1,t} + k_1 \cdot x_{4,t} - x_{2,t} - x_{5,t} + \sqrt{2 + 2k_1^2} \cdot r < 0 \qquad (26)$$

$$x_{2,t} + x_{5,t} - k_2 \cdot x_{1,t} - k_2 \cdot x_{4,t} + \sqrt{2 + 2k_1^2} \cdot r < 0 \qquad (27)$$

$$-x_{2,t} - x_{5,t} + n \cdot \sum_{j=1}^{9} x_{j,t} + \sqrt{2 + 9 \cdot n^2} \cdot r < 0 \qquad (28)$$

$$z_1 \cdot x_{1,t} - x_{4,t} + \sqrt{z_1^2 + 1} \cdot r < 0 \quad (29)$$

$$x_{4,t} - z_2 \cdot x_{1,t} + \sqrt{1 + z_2^2} \cdot r < 0 \quad (30)$$

$$w_1 \cdot x_{3,t} - x_{4,t} + \sqrt{w_1^2 + 1} \cdot r < 0 \quad (31)$$

$$x_{4,t} - w_2 \cdot x_{3,t} + \sqrt{1 + w_2^2} \cdot r < 0 \quad (32)$$

$$v_1 \cdot x_{2,t} - x_{5,t} + \sqrt{v_1^2 + 1} \cdot r < 0 \quad (33)$$

$$x_{5,t} - v_2 \cdot x_{2,t} + \sqrt{1 + v_2^2} \cdot r < 0 \quad (34)$$

$$x_{j+33,t} + x_{j+42,t} - \sqrt{2} \cdot r > h_{j,t}\ ,\ j = \overline{1,9} \quad (35)$$

$$x_{j+33,t} + x_{j+42,t} + \sqrt{2} \cdot r < H_{j,t},\ j = \overline{1,9} \quad (36)$$

$$x_{j+51,t} - r > m_{j,t},\ j = \overline{1,9} \quad (37)$$

$$x_{j+51,t} + r < M_{j,t},\ j = \overline{1,9} \quad (38)$$

$$x_{1,t} + x_{2,t} + x_{3,t} + x_{4,t} + x_{5,t} + x_{6,t} + x_{7,t} + x_{8,t} + x_{9,t} + \sqrt{9} \cdot r < L_t \quad (39)$$

$$x_{j,t} - r > 0,\ \ j = \overline{1,9} \quad (40)$$

$$\Sigma_{j=1}^{9} x_{j,t} \cdot q_{j,k} \cdot p_{j,k} - \sqrt{\Sigma_{j=1}^{9} q_{j,t}^2 \cdot p_{j,t}^2} \cdot r > \gamma_0 \quad (41)$$

We introduce the following variables in table 1.

Table 1: Variables of the model.

№	Age and sex groups of the flock	The sheep at the beginning of the year	Flock of Income			Flock of Expenses				Sheep at the end of the year
			lamb	aging sheep	purchase d sheep	sheep to be moved	sheep for sale	sheep for laughter	of bnormal loss	
1	Ram	$b_{1,t}$		$x_{10,t}$	$x_{16,t}$	$x_{25,t}$	$x_{34,t}$	$x_{43,t}$	$x_{52,t}$	$x_{1,t}$
2	Ewe	$b_{2,t}$		$x_{11,t}$	$x_{17,t}$	$x_{26,t}$	$x_{35,t}$	$x_{44,t}$	$x_{53,t}$	$x_{2,t}$
3	Male sheep	$b_{3,t}$		$x_{12,t}$	$x_{18,t}$	$x_{27,t}$	$x_{36,t}$	$x_{45,t}$	$x_{54,t}$	$x_{3,t}$
4	Year-old- ram	$b_{4,t}$		$x_{13,t}$	$x_{19,t}$	$x_{28,t}$	$x_{37,t}$	$x_{46,t}$	$x_{55,t}$	$x_{4,t}$
5	Female- year-old lamb	$b_{5,t}$		$x_{14,t}$	$x_{20,t}$	$x_{29,t}$	$x_{38,t}$	$x_{47,t}$	$x_{56,t}$	$x_{5,t}$
6	Male-year- old lamb	$b_{6,t}$		$x_{15,t}$	$x_{21,t}$	$x_{30,t}$	$x_{39,t}$	$x_{48,t}$	$x_{57,t}$	$x_{6,t}$
7	Lamb-ram	$b_{7,t}$	$n_{1,t}$		$x_{22,t}$	$x_{31,t}$	$x_{40,t}$	$x_{49,t}$	$x_{58,t}$	$x_{7,t}$
8	Felame lamb	$b_{8,t}$	$n_{2,t}$		$x_{23,t}$	$x_{32,t}$	$x_{41,t}$	$x_{50,t}$	$x_{59,t}$	$x_{8,t}$
9	Male lamb	$b_{9,t}$	$n_{3,t}$		$x_{24,t}$	$x_{33,t}$	$x_{42,t}$	$x_{51,t}$	$x_{60,t}$	$x_{9,t}$

In order to solve a problem (20)-(41), we use livestock growth of Zavkhan province computed by stochastic and Exponential and Logistic Weighted Sum (ELWS) method in [6].

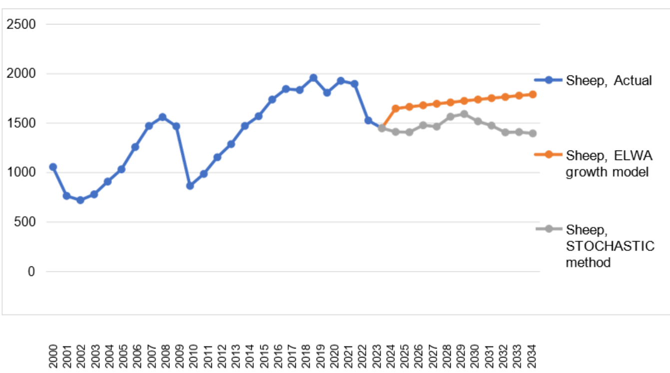

Figure 1: Actual and prospective trends in sheep herd size calculated using ELWS and stochastic methods. (thous.heads)

Source: National Statistics Committee, www.1212.mn, Researcher's calculation

Table 2: Results of calculating the prospective trends of the sheep herd up to 2035 using the Exponential–Logistic–Weighted Average method, and the projected herd structure. (thous.heads)

Year	The number of sheep	Ram	Ewe	Male sheep	Year old ram	Year- old female lamb	Year- old male lamb	Lamb ram	Female lamb	Male lamb
2024	1442.62	5.99	484.90	254.19	0.08	137.00	143.74	0.08	200.55	216.09
2025	1640.76	6.82	551.50	289.10	0.09	155.81	163.48	0.09	228.10	245.77
2026	1656.95	6.88	556.94	291.95	0.09	157.35	165.10	0.09	230.35	248.19
2027	1672.65	6.95	562.22	294.72	0.10	158.84	166.66	0.09	232.53	250.55
2028	1687.88	7.01	567.34	297.40	0.10	160.29	168.18	0.09	234.65	252.83
2029	1702.63	7.07	572.30	300.00	0.10	161.69	169.65	0.09	236.70	255.04
2030	1716.93	7.13	577.10	302.52	0.10	163.05	171.07	0.09	238.69	257.18
2031	1730.78	7.19	581.76	304.96	0.10	164.36	172.45	0.09	240.61	259.25
2032	1744.20	7.25	586.27	307.33	0.10	165.64	173.79	0.09	242.48	261.26
2033	1757.20	7.30	590.64	309.62	0.10	166.87	175.09	0.09	244.29	263.21
2034	1769.81	7.35	594.88	311.84	0.10	168.07	176.34	0.09	246.04	265.10
2035	1782.03	7.40	598.98	313.99	0.10	169.23	177.56	0.09	247.74	266.93
Structure, %	100	0.42	33.61	17.62	0.01	9.50	9.96	0.01	13.90	14.98

Source: Researcher's calculation

Table 3: Results of stochastic estimation of the prospective trends of the sheep herd up to 2035 and the projected herd structure

Year	The number of sheep, calculation	Ram	Ewe	Male sheep	Year old ram	Year- old female lamb	Year- old male lamb	Lamb ram	Female lamb	Male lamb
2024	1442.62	5.99	484.90	254.19	0.08	137.00	143.74	0.08	200.55	216.09
2025	1404.39	5.83	472.05	247.45	0.08	133.37	139.93	0.07	195.24	210.36
2026	1400.24	5.82	470.66	246.72	0.08	132.97	139.52	0.07	194.66	209.74
2027	1470.74	6.11	494.35	259.14	0.08	139.67	146.54	0.08	204.46	220.30
2028	1457.07	6.05	489.76	256.74	0.08	138.37	145.18	0.08	202.56	218.25
2029	1556.65	6.47	523.23	274.28	0.09	147.83	155.10	0.08	216.41	233.17
2030	1585.01	6.58	532.76	279.28	0.09	150.52	157.93	0.08	220.35	237.42
2031	1511.47	6.28	508.04	266.32	0.09	143.54	150.60	0.08	210.12	226.40
2032	1467.97	6.10	493.42	258.65	0.08	139.41	146.27	0.08	204.08	219.89
2033	1398.60	5.81	470.11	246.43	0.08	132.82	139.35	0.07	194.43	209.50
2034	1402.23	5.82	471.32	247.07	0.08	133.16	139.72	0.07	194.94	210.04
2035	1389.27	5.77	466.97	244.79	0.08	131.93	138.42	0.07	193.14	208.10
Structure, %	100.00	0.42	33.61	17.62	0.01	9.50	9.96	0.01	13.90	14.98
Source: Calculated based on the average herd structure for 2020-2023										

Source: Researcher's calculation

Due to the influence of external and internal factors, livestock numbers may decline, offspring survival rates may decrease, livestock morbidity may increase, and herd structure may deteriorate. As a result of fluctuations in livestock numbers, income derived from the herd tends to be unstable.

The age- and sex-structured composition of the sheep flock for the period 2025–2035 was derived using Sphere Packing Theory. Problem (20)-(41) was solved on Matlab. In numerical computations, we have taken γ_0 as 70 billion MNT.

Results of Computations

Table 4: Optimal structure of sheep for 2025, 2030 and 2035 (thous.heads)

2025 Year										
Age and sex groups of the flock	i	Stochastic method,	Low bound of x_i	r*=3.882	Upper bound of x_i		ELWS method	Low bound of x_i	r*=4.535	Upper bound of x_i
				Solution, x_i					Solution, x_i	
Ram	1	5.83	8.42	9.72	11.01		6.82	9.84	11.35	12.86
Ewe	2	472.05	474.64	475.93	477.23		551.50	554.52	556.03	557.55
Male sheep	3	247.45	250.04	251.33	252.63		289.10	292.12	293.64	295.15
Year old ram	4	0.08	2.67	3.96	5.26		0.09	3.12	4.63	6.14
Year-old female lamb	5	133.37	97.85	99.14	100.44		155.81	114.32	115.83	117.34
Year-old male lamb	6	139.93	142.52	143.81	145.11		163.48	166.51	168.02	169.53
Lamb ram	7	0.07	54.63	55.93	57.22		0.09	63.83	65.34	66.85
Female lamb	8	195.24	183.15	184.44	185.73		228.10	213.97	215.48	217.00
Male lamb	9	210.36	124.02	125.32	126.61		245.77	144.90	146.41	147.92
Number of sheeps		1404.39	1337.94	1349.58	1361.23		1640.76	1563.12	1576.73	1590.33
2030 Year										
	i	Stochastic method,	Low bound of x_i	r*=4.381	Upper bound of x_i		ELWS method	Low bound of x_i	r*=2.538	Upper bound of x_i
				Solution, x_i					Solution, x_i	
Ram	1	6.58	9.50	10.97	12.43		7.13	8.35	8.68	9.00
Ewe	2	532.76	535.68	537.14	538.60		577.10	544.66	544.99	545.31
Male sheep	3	279.28	282.20	283.66	285.12		302.52	222.16	222.48	222.81
Year old ram	4	0.09	3.01	4.47	5.93		0.10	0.26	0.59	0.92
Year -old female lamb	5	150.52	110.44	111.90	113.36		163.05	84.33	84.66	84.99
Year -old male lamb	6	157.93	160.85	162.31	163.77		171.07	161.22	161.55	161.87
Lamb ram	7	0.08	61.66	63.12	64.58		0.09	63.58	63.91	64.24
Female lamb	8	220.35	206.70	208.16	209.62		238.69	215.13	215.46	215.79
Male lamb	9	237.42	139.97	141.43	142.89		257.18	145.41	145.74	146.06
Number of sheeps		1585.01	1510.02	1523.16	1536.30		1716.93	1445.10	1448.05	1451.00
2035 Year										
	i	Stochastic method,	Low bound of x_i	r*=4.840	Upper bound of x_i		ELWS method	Low bound of x_i	r*=4.925	
				Solution, x_i					Solution, x_i	
Ram	1	5.77	8.33	9.61	10.89		7.40	10.69	12.33	13.97
Ewe	2	466.97	469.53	470.81	472.09		598.98	602.27	603.91	605.55
Male sheep	3	244.79	247.35	248.63	249.91		313.99	317.28	318.92	320.56
Year old ram	4	0.08	2.64	3.92	5.20		0.10	3.38	5.03	6.67
Year -old female lamb	5	131.93	96.80	98.08	99.36		169.23	124.16	182.48	127.45
Year -old male lamb	6	138.42	140.98	142.26	143.54		177.56	180.84	70.96	184.13
Lamb ram	7	0.07	54.04	55.32	56.60		0.09	69.32	70.96	72.60
Female lamb	8	193.14	181.18	182.46	183.74		247.74	232.39	234.04	235.68
Male lamb	9	208.10	122.69	123.97	125.25		266.93	157.37	159.01	160.66
Number of sheeps		1389.27	1323.53	1335.05	1346.57		1782.03	1697.71	1712.48	1727.26

Source: Researcher's calculation

For each projection scenario, the application of Sphere Packing Theory provides a basis for stabilizing flock productivity and the associated income streams.

Conclusion

We determined optimal structure of livestock for case of sheep population of Zavkhan province of Mongolia, using sphere packing theory. In numerical calculations, we used sheep growth previously computed in [5]. Sphere packing theory allows us to find optimal range of number of sheep for deferent age, ensuring a given level of profit.

The application of sphere packing theory in livestock husbandry represents a new approach to developing a sustainable animal production [7,8].

The proposed method and its application can be used for other types of livestock such as house, goat, camel and cow [9].

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